

### **Q11. Partitions of a Number (60 Marks):**

Given a positive integer  $N$ , represented in the decimal system, it can be expressed as an unordered sum of positive integers, where each of the summands can be expressed as  $2^a \times 3^b$ , and  $a$  and  $b \geq 0$ . For example, 3 can be partitioned in three distinct ways:

$$\begin{aligned}3 &= 3 = 2^0 \times 3^1 \\3 &= 2 + 1 = 2^1 \times 3^0 + 2^0 \times 3^0 \\3 &= 1 + 1 + 1 = 2^0 \times 3^0 + 2^0 \times 3^0 + 2^0 \times 3^0\end{aligned}$$

Note that the order of the summands does not matter. For example, the partitions of  $3 = 2 + 1$  and  $3 = 1 + 2$  are considered the same partition.

We further define a **VALID** partition as follows. A partition is considered to be **VALID** if the following conditions are fulfilled:

- 1) None of the summands can divide any of the other summands.  
For example, the partition of  $3 = 2 + 1$  is **INVALID** since 2 is divisible by 1.
- 2) The only **VALID** partition of 3 is  $3 = 2^0 \times 3^1$ .

Given an integer,  $N$ , find the number of **VALID** partitions of  $N$ . For example,

- If  $N = 11$ , the **VALID** partitions are
  - a.  $2 + 9$ , which is  $2^1 \times 3^0 + 2^0 \times 3^2$
  - b.  $8 + 3$ , which is  $2^3 \times 3^0 + 2^0 \times 3^1$So the number of **VALID** partition(s) of 11 is 2.
- If  $N = 17$ , the only **VALID** partition is
  - a.  $8 + 9$ , which is  $2^3 \times 3^0 + 2^0 \times 3^2$So the number of **VALID** partition(s) of 17 is 1.

### **Write a programme to**

**Input** an integer  $N$ , where  $1 \leq N \leq 100000$ .

**Output** the number of **VALID** partition(s) of  $N$ .

### 试题 11. 数字的分拆 (60 分) :

给定一个十进位制的正整数  $N$ , 它可以分拆成若干个正整数之和。我们进一步要求每个加数都必须表达为 2 和 3 的乘积, 即  $2^a \times 3^b$ , 其中  $a$  和  $b \geq 0$ 。例如, 3 有三个不同的分拆方式:

$$\begin{aligned}3 &= 3 = 2^0 \times 3^1 \\3 &= 2 + 1 = 2^1 \times 3^0 + 2^0 \times 3^0 \\3 &= 1 + 1 + 1 = 2^0 \times 3^0 + 2^0 \times 3^0 + 2^0 \times 3^0\end{aligned}$$

必须注意的是, 加数的排列顺序并不重要。例如,  $3 = 2 + 1$  和  $3 = 1 + 2$  是相同的分拆方法。

一个数字可以有很多分拆的方法。在这一试题中, 我们对一些特定的分拆感兴趣。我们定义一个**有效**的分拆, 必须满足以下条件:

- (1) 任一加数不能被另一个加数整除。  
例如, 3 的其中一个分拆,  $3 = 2 + 1$  是**无效**的, 因为 2 能被 1 整除。
- (2) 唯一**有效**分拆 3 的方法是  $3 = 2^0 \times 3^1$ 。

给定一个正整数,  $N$ , 找出能够**有效**分拆  $N$  的方法的总数。例如:

- 假设  $N = 11$ , **有效**的分拆为
  - a.  $2 + 9$ , 即  $2^1 \times 3^0 + 2^0 \times 3^2$
  - b.  $8 + 3$ , 即  $2^3 \times 3^0 + 2^0 \times 3^1$所以能够**有效**分拆 11 的方法总共有 2 种。
- 假设  $N = 17$ , **有效**的分拆为
  - a.  $8 + 9$ , 即  $2^3 \times 3^0 + 2^0 \times 3^2$所以能够**有效**分拆 17 的方法总共有 1 种。

#### 试写一程式以

**输入** 一个正整数  $N$ ; 已知  $N$  满足条件  $1 \leq N \leq 100000$ 。

**输出** 能够**有效**分拆  $N$  的方法的总数。

**Example (例子)**

<b>Input (输入)</b>	<b>Output (输出)</b>
100000	11
5938	6
99	2
21112	9
434	1